Numerical methodology for modulus of elasticity calculating of round wooden beams: *Pinus elliottii*
ABSTRACT

Due to its abundance and versatility, wood always has been considered an excellent structural material. In view of this, several studies involving wood have been developed, in order to deepen knowledge and improve its structural design. Thus, this work aims to present a numerical calculation methodology for obtaining the longitudinal modulus of elasticity, focusing only on round beams. For this, Finite Element Method and Least Squares Method were used, considering the influence of irregularities in pieces geometry by linear approximations along the finite elements. In addition to these two methods, an analytical numerical model (simplified 1D) was used, which considers a single diameter value and a numerical one (3D), which allows the round structural elements geometry to be delineated. To obtain modulus of elasticity, it was adopted the three-point static bending test, using 24 round beams of *Pinus elliotti*. Confidence interval of the difference between longitudinal modulus of elasticity obtained by the numerical model 1D ($E_o$) and analytical procedure ($E_{deq}$) revealed statistical equivalence, once small conicity were observed in the tested round beams. Using $E_o$ in simulations of plumb elements in three dimensions led to equivalent displacements, proving the numerical model 1D efficiency, showing that the latter is more interesting because it requires less computational effort.

**Keywords:** Round Wood Beams, Finite Element Method, Least Squares Method.
INTRODUCTION

Due to its versatility and availability, for centuries, wood has been employed as a structural material. In countries with a tradition of using wood in structures, it is common to adopt mixed systems, both of solid wood and its based products. However, demands associated with processing costs motivate research to find solutions that combine high wood efficiency as a structural element at a low production cost. An alternative to solve this problem is applying this material in its original plump form, due to the natural tree growth (PARTEL, 1999; ZANGIÁCOMO; CHRISTOFORO; LAHR, 2013).

Research in order to expand the range of technical and scientific knowledge related to roundwood has been developed by scientists in different world regions. Cerda and Wolfe (2003) evaluated the conventional bending strength (modulus of rupture) in 45 Pinus radiata round beams species, with an average length of 12 meters and without knots over 25 mm. Samples were subjected to static bending tests following ASTM D 1036 (1998). It was observed that the results obtained were similar to those of other Pinus species, concluding that the beams in this study can be included in the ASNI 05.1 group of standard specification for timber beams.

Larson et al. (2004) evaluated strength and stiffness properties of round pieces of Pinus ponderosa from the static bending test standardized by ASTM D 198 (1997). Among the results obtained, authors argued that round beams bending strength showed successive reductions with the increase in the parts diameters.

Green et al. (2004) evaluated strength and stiffness of Abies lasiocarpa and Pinus contorta round pieces, with an average length of 4.9 m and diameter close to 23 cm. Logs were submitted to the static bending and compression parallel to grain tests, following premises of the normative document ASTM D 198 (2003). Modulus of elasticity values were obtained both by methods of transverse vibration and stress wave, showing good correlations with modulus of rupture.

Miná et al. (2004) evaluated strength and stiffness properties of round posts with the aid of three-point static bending test and ultrasound. For samples characterization, a total of fifty posts of Corymbia citriodora were used: fifteen with 7.5 m length and 210 mm average diameter; and thirty five with 9 m length and 135 mm average diameter. After being tested by ultrasound method, the specimens were mechanically tested, for determining static and dynamic modules of elasticity. After results analysis, good correlations were observed between the mechanical properties calculated by both methods. This allowed concluding that ultrasound method can be applied to wooden poles aiming to determine their stiffness.

Green et al. (2006) evaluated strength and stiffness properties in round wood. For this purpose, one hundred ten trees were selected: forty two of which Alpine fir (Abies lasiocarpa);
forty of Lodgepole pine (*Pinus contorta*); twenty three of Engelmann spruce (*Picea engelmannii*); two of Western white pine (*Pinus monticola*) and one of Grand fir (*Abies grandis*). The trees were processed into specimens with 2.28 cm diameter, and length ranging from 4.3 to 4.9 m. Samples were subjected to tests of transverse vibration, three-point static flexion (ASTM D 198, 1997) and compression parallel to grain, thus obtaining dynamic and static modulus of elasticity. The dynamic elasticity modules showed to be close to the static ones due to the good correlations found.

Fernández-Golfín *et al.* (2007) performed mechanical classification on round wood of *Pinus nigra*. For this, four hundred forty-five pieces, diameters of 8; 10; 12 and 14 cm, and lengths of 2; 2.5 and 3m were employed. Specimens were subjected to mechanical tests to the point of failure, as recommended by the European standard EN 14251 (2004), obtaining values of modulus of rupture and modulus of elasticity.

Green *et al.* (2008) evaluated strength and stiffness in bending and in compression parallel to grain of round wood, according to static and dynamic calculation approaches, using one hundred sixty trees of *Pseudotsuga menziesii* and two hundred eighteen of *Pinus ponderosa*. The tested beams dimensions were 4 m length and diameter ranging from 10 to 20 cm. From the static approach, elasticity and rupture modules were obtained following calculation proposed by ASTM D 198 (2003). Dynamic modules of elasticity were calculated by transversal vibration. Data analysis showed good correlations between static and dynamic elasticity modules, and between strengths.

Zangiácomo and Lahr (2008) studied the relationship between span (L) and diameter (*d*<sub>eq</sub>) in round wood beams for which the effect of shear stress becomes negligible on displacements, reaching L/*d*<sub>eq</sub> ratios of 12 and 15 for *Pinus elliottii* and *Pinus caribaea*, respectively.

Morgado *et al.* (2009) evaluated the mechanical properties of round pieces of *Pinus pinaster*, using five hundred pieces with structural dimensions. A new selection was made based on pieces diameter, knot diameter, growth rings width and biological degradation signs, leaving only two hundred pieces with diameters ranging from 7 to 9 cm. Modules of elasticity and rupture in bending were obtained using European standard BS EN408 (2003), adapted for round sections. Results showed good correlations between both investigated properties.

Vestol and Hoibo (2010) evaluated the density, modulus of rupture and elasticity of round pieces of *Pinus sylvestris*, using five hundred thirty-three samples, diameters ranging from 7.5 to 25 cm and length from 2.1 to 6.1 m. Those properties were obtained according to the standard EN 14251 (2004). The authors conclude, among others, that the plump woods may present values of modulus of rupture and elasticity different depending on the trees extraction region.
Christoforo et al. (2011) evaluated position influence of round structural pieces of *Eucalyptus grandis* on modulus of elasticity in static bending calculation, using twenty-four pieces, 7.5 m length, and 30 cm average diameter. Modulus of elasticity were obtained according to ABNT NBR 7190 (1997) – Timber Structures Design, adapted for structural pieces and for non-destructive tests. The moment of inertia used to calculate the modulus of elasticity was the equivalent, obtained at the midpoint. The change in the parts position in the bending test led to different stiffness values, concluding that it is important that the bending test is performed considering at least two different positions (rotations around its axis).

With the increasing use of round wood for the most different building and structural applications, several countries have been busy planning plantings and cutting fast-growing trees to supply the demand for such products. This is the case of Latvia, a Baltic country where wood is traditionally used in civil construction, as pointed out by Krummis et al. (2012). In the work presented, authors highlight the relevance of encouraging expansion of planting areas, avoiding interruption of the supply of this important input.

In general, research involving the use of mechanical tests to obtain strength and stiffness properties in rounded pieces of wood makes use of normative documents (ANSI O5.1, 2002; ASTM D198, 1997; ASTM D1036, 1998; DIN EN 14251, 2004; AS 2209, 1994; ABNT NBR 6231, 1980; ABNT NBR 8456, 1984; ABNT NBR 8457, 1984; ABNT NBR 6122, 1996), which consider the part's geometry to be perfectly conical, leading to simplified equations for calculating modulus of elasticity in static bending.

The classification of round pieces of wood depends on different variables, such as defects and other morphological characteristics of a tree trunk. Different species, whose trees grow in different forest sites, and in different edaphic conditions, present a high probability of having their wood influenced by such factors, as pointed out by Karaszewski et al. (2013). These authors determined the distribution of quality classes of wood thus produced, studying about 1400 pieces of Beech, from which it was possible to indicate the most suitable applications.

Joining round pieces segments for compose structural elements of greater length can be an efficient method for converting small diameter material, harvested during roughing operations, into value-added products. Study by Piao et al. (2013) investigated mechanical performance of finger-joints for connecting round pieces. It was observed that strength and stiffness properties, and deformation variation of the pieces obtained by this kind of structural union were affected, showing the need to continue in this line of research, aiming to reach better use of material, previously discarded.

Considering it is necessary to extend service life of raw material applied in timber bridges, Plaxico et al. (2015) present results of their study, developed a procedure for measuring...
the amount of deterioration damage of round wood. In specific approach, guardrail posts were extracted from damaged installations in Ohio, USA. A resistograph device was used for measures. Deterioration damage levels ranged from severe to essentially undamaged. Authors concluded the adopted procedure is a good alternative to determine quantitative damage score and confirmed importance of pre-treat round wood for outdoor application.

Yet in relation to pre-treated round wood, Kang et al. (2017) investigated effect of polyethylene glycol (PEG) treatment in logs of Red Pine and Korean Pine, in order to apply them in house construction. Logs used for this study were impregnated with a solution of PEG-1000 for three weeks, and then they were dried to moisture content close to 15% at 120°C temperature, within 49 h drying time. Results showed reduction of logs surface damage and supported conclusion that the proposed procedure can efficiently contribute to extend material service life so treated.

Finally, and based on necessity of ensure constant offer of round wood, Hernandez (2019) developed models to analyzing supply chains as regards pattern of consumption and production. These models are considered useful for availability estimations of natural resources, as forest wood supply chains. This work presents an analysis of a forest wood supply chain focusing on forest operations to estimate round-wood volume availability, and their technical constraints, using a local case study in Mexico. Conclusion pointed out to be possible to extend application of this model for every country around world.

### OBJECTIVE

This work aims to present a numerical calculation methodology, based on the Finite Element Method (numerical 1D) and the Least Squares Method to obtain modulus of elasticity in static bending, in round wood structural pieces, considering the existing irregularities of their shapes (geometry). For this purpose, the three-point static flexion test is used, together with the numerical model 1D, simplified, (analytical - 1D), which considers circular section constant for the part; and another numeric one (3D), allowing the numerical models to approximate the trunk-conical geometry part by parts, allowing to evaluate the approximation precision contained in the simplified analytical model.
METHODS

To determine the modulus of elasticity in static bending, twenty-four round structural pieces of Pinus elliottii were used, with an average apparent density of 0.57 g/cm³; 6.3 m average length; 21 cm average diameter; green wood (saturated), 6% average taper; and 0.82 average shape factor of the cross section of 0.82. Tests were carried out at Wood and Timber Structures Laboratory (LaMEM), Department of Structural Engineering (SET), São Carlos School of Engineering (EESC), University of São Paulo (USP).

Round pieces met $L/d_{eq}>15$ ratio, $L$ being the span and $d_{eq}$ the equivalent diameter (ZANGIÁCOMO; LAHR, 2008) measured at the midpoint, assuming that: sections are perfectly circular; diameters vary linearly in length; maximum displacement occurs at the point of force application.

The modulus of elasticity in this work is evaluated according to two different mathematical models of calculation, both using the structural scheme of three-point static bending. For the first case (Figure 1), the modulus of elasticity was calculated using Equation 1.

\[ E = \frac{4 \cdot F \cdot L^3}{3 \cdot \pi \cdot \delta \cdot d_{eq}^4} \]  

Figure 1. Three-point static bending test and equivalent diameter ($d_{eq}$) location.

Where: $E$ is the modulus of elasticity in static bending; $F$ is the applied force; $L$ is the span; $d_{eq}$ the equivalent diameter; and $\delta$ is the displacement measured under the point of force application.

For the second case, as an alternative form of calculation, it is proposed that the effective modulus of elasticity ($E_{o}$) value be calculated according to the structural test scheme illustrated by Figure 2, with seven deflectometers (R).
In determining the effective elastic modulus (numeric 1D) for each structural test performed, seven deflectometers were placed along the parts, L/8 apart from each other, and also nine circumference values were measured along its length for determining their respective diameters (d).

Displacements read in deflectometers 1 to 7 (Figure 2) were taken when the displacement in the middle of the span is L/200 (L expressed in centimeters). This value guarantees material elastic-linear behavior and geometry linearity for the part, being a measure of small displacements (ABNT NBR 7190, 1997). The imposition of this relationship allows static bending tests to be performed in a non-destructive way (CHRISTOFORO et al., 2011; ZANGIÁCOMO; CHRISTOFORO; LAHR, 2013).

For the effective modulus of elasticity (E_o) calculation, a computer program “effective elastic modulus” (E_{otm}) was developed on Finite Element Method (FEM) basis, according to the kinematic model of Bernoulli’s beam deformation, in Principle of Virtual Works (PTW), disregarding, in these calculations, the forces per volume unit and the taper effects.

The bar finite element presents two degrees of freedom per node, two translations and two rotations, formulated using a third-degree interpolative polynomial function (CHRISTOFORO et al., 2012).

The approximate geometry for the part is considered linear by parts. For each two successive circumferences (Figure 2), a finite element of trunk-conical variation is used and the cross-section moment of inertia is defined by Equation 2.

\[
I(x) = \frac{\pi}{4} \left( \frac{r_f - r_i}{h_e} \cdot x + r_i \right)^4
\]  

(2)

Where: I(x) is the moment of inertia; r_i and r_f the radius calculated from the measurements of two successive circles; h is the finite element length; and x the real number in the range \([0, h_e]\).
The experimental displacements values, as well as the diameters values measured along the element, were supplied to the $E_{\text{otm}}$ program, in order to calculate the effective value of the part’s modulus of elasticity.

The $E_{\text{otm}}$ program, based on FEM fundamentals, determines a displacement vector (numeric), accounting for the influence of existing irregularities in the part geometry, having the structural element modulus of elasticity as a dependent variable.

With the vector of displacements ($U^{(n)}$) determined by the program, and the vector of experimental displacements ($U^{(e)}$), a function ($f(E)$) is constructed, based on the Least Squares Method (Equation 3), whose objective is to determine the modulus of elasticity value so that the residue generated by both solutions, numerical and experimental, is minimal, obtained by Newton-Raphson Method.

$$f(E) = \frac{1}{2} \sum_{i=1}^{n} \left( U^{(e)}_i - U^{(n)}_i \right)^2$$ (3)

Alternatively, after calculating the effective modulus of elasticity (numerical 1D), these values, together with those relating to geometry, boundary conditions, strength and linkages, were used for the numerical simulation of parts with three-dimensional finite elements (3D numeric), aiming to reproduce the experimental displacements, in order to check the possible differences.

The simulations were performed using ANSYS14.0 - Workbench software, with the finite element MESH200. It is noteworthy that the round pieces, as is the case here evaluated, can be treated as isotropic material (CHRISTOFORO et al., 2011; ZANGIÁCOMO; CHRISTOFORO; LAHR, 2013). Due to the lack of information on wood anisotropy in the normative document NBR 7190 (ABNT, 1997), it was decided to consider the Poisson’s ratio to be null.

In order to verify the existing differences between the calculated modulus of elasticity considering the simplified model ($E_{\text{deq}}$) and the proposed alternative calculation methodology ($E_o$), the confidence interval of the difference between two averages was used, expressed by Equation 4.

$$\bar{x}_m - t_{\alpha/2,n-1} \cdot \frac{S_m}{\sqrt{n}} \leq \mu \leq \bar{x}_m + t_{\alpha/2,n-1} \cdot \frac{S_m}{\sqrt{n}}$$ (4)

Where: $\mu$ is the differences population mean; $\bar{x}_m$ is the differences sample arithmetic mean, in the sample size; $S_m$ is the differences sample standard deviation; and $t_{\alpha/2,n-1}$ is
the tabulated value of the distribution of Student (t) with n-1 degrees of freedom and significance level $\alpha$.

### RESULTS

The modules values $E_{deq}$ and $E_o$ obtained for the round structural pieces of *Pinus elliottii* are presented in Table 1, with DP being the standard deviation and CV the variation coefficient.

**Table 1.** Modules of elasticity values obtained for the round wood.

<table>
<thead>
<tr>
<th>Piece</th>
<th>$E_{deq}$ (MPa)</th>
<th>$E_o$ (MPa)</th>
<th>Piece</th>
<th>$E_{deq}$ (MPa)</th>
<th>$E_o$ (MPa)</th>
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<td>11631</td>
<td>12382</td>
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<td>8723</td>
<td>24</td>
<td>10904</td>
<td>10349</td>
</tr>
</tbody>
</table>

Average $9342$ $9363$

DP $2116$ $2099$

CV (%) $23$ $22$

Figure 3. shows the Anderson-Darling normality test results for modules of elasticity, Table 1.

![Figure 3. Normality tests.](image)

The confidence interval found between $E_{deq}$ and $E_o$ is $1246 \leq \mu \leq 1205$ and, since zero belongs to the subset, it appears that these are statistically equivalent.

The linear regression between $E_{deq}$ and $E_o$ values is illustrated by Figure 4.
Figure 4. Linear regression between $E_{deq}$ and $E_o$.

For simulations with three-dimensional finite elements, initially, the mesh influence on displacement calculation was evaluated. For this purpose, the test data from part 1 (Table 1) were used, being: $E_o = 9111$ MPa; $F = 5$ kN, $L = 5700$ mm; and diameters ($d$): 206 mm (base); 217 mm; 206 mm; 198 mm; 194 mm; 191 mm; 178 mm; 185 mm; 181 mm (top). Finite elements were defined with edge sizes 10 and 20 mm. Generated meshes can be seen in Figures 5a and 5b, respectively.

Figure 5. Part discretization according to the finite element meshes.

Figures 6a and 6b illustrate the structural part displacements as a meshes function used with 10 and 20 mm edges, respectively.
DISCUSSION

The structural part simulations with finite elements with 10 mm edge led to a mesh with about 399,116 elements, with 60,920 being the number of elements with a 2 mm edge. The relative percentage difference between displacements at the midpoint for both meshes was 0.22%. For this reason, all simulations were performed with the finite element of 20 mm edge.

The experimental displacement measured at the point of force application (span midpoint) of part 1 was 28.50 mm, while the numerical displacement at the same point was 30.36 mm, giving a relative percentage error of 6.53% between them. In all parts, when comparing experimental displacements with the respective displacements of three-dimensional numerical model, the greatest percentage differences always occurred at the midpoint. This error in the twenty-four simulated pieces ranged from 4.36% to 7.49%. Confidence intervals result between numerical model displacements and the respective of experimental per piece proved statistical equivalence in all cases.

CONCLUSION

Confidence interval between the longitudinal modulus of elasticity obtained from numerical approach (1D) proposed here, which takes into account shape irregularities, with the analytical (simplified 1D), proved statistical equivalence between both, revealing the good approximation of the simplified model, calculation methodology present in national and international Codes, essentially justified by small average taper found in the plump elements of *Pinus elliottii* tested here. Results may be different for wood with different taper, thus justifying the methodology proposal use.

The modulus of elasticity of numerical approach of calculation ($E_o$) used in the plump elements simulations in three dimensions, carried out with the commercial software ANSYS14.0
aid, made it possible to verify the numerical displacements proximity with those obtained from experiments, and statistical equivalence piece by piece using the confidence interval. Due to small pieces conicity, which resulted in the equivalence between the modules of elasticity, in this case, the pieces could be simulated in three dimensions with the use of $E_{deq}$, leading to results equivalent to those obtained with the use of $E_o$.

However, for other conicities, the results may be different. In general, the simulation results in three dimensions for the displacements confirm, as expected by theory, in their equivalence with the numerical model 1D, showing that the latter is more interesting because it requires less computational effort.

The joint use of both forms of calculation presented here (simplified 1D and numerical 1D) is shown as an alternative classification methodology for the round wood element. If longitudinal modulus of elasticity results for both calculation methodologies are statistically equivalent, this implies that the part has a small taper and regular geometry, otherwise, the taper and irregularities in the shape must be taken into account, and the part modulus of elasticity value must be obtained using the numerical calculation strategy.

### REFERÊNCIAS


